Probability Density Function Modeling for Sub-Powered Interconnects

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This paper proposes three mathematical models for reliability probability density function modeling the interconnect supplied at sub-threshold voltages: spline curve approximations, Gaussian models,and sine interpolation. The proposed analysis aims at determining the most appropriate fitting for the switching delay – probability of correct switching for sub-powered interconnects. We compare the three mathematical models with the Monte-Carlo simulations of interconnects for 45 nm CMOS technology supplied at 0.25V.

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RELIABILITY ISSUES OF INTERCONNECTS

One of the most important issues in the today's nanometer CMOS devices is represented by their low reliability. One of the most important causes of the poor reliability is related to the process variations, which means that the physical attributes of both transistors and the wires connecting them vary for different components. These issues are further augmented by the aggressive voltage scaling performed in order to address the power consumption issues in the modern digital circuits. Due to the process and supply voltage variations, as well as the very low supply voltages, equal or smaller than the transistors threshold voltages, CMOS circuits present a probabilistic behavior.

Regarding the interconnects, the most important reliability factors are represented by crosstalk and processed variations. Crosstalk (capacitive and inductive) represents the effect of the neighboring wires having on a target wire [9]. Capacitive crosstalk is due to the capacitive coupling between the two neighboring wires. Inductive crosstalk manifests over multiple wires. In today's nanometer technology, the capacitive effect is the dominant component of the crosstalk. Crosstalk manifests in two ways: it may determine a glitch on a static wire (a wire which does not switches) or it may affect the delay characteristics of switching wires. In both cases, the crosstalk is data dependent. The crosstalk effects on wire are aggravated by the process variations. For interconnects, these are due: device geometry variations, device material and electrical parameter variations, interconnect geometry and material parameter variations [1, 2, 6]. These types of variations affect the electrical measures of the interconnect - resistance, capacitance or inductance. Due to these variations, both the crosstalk effect and the timing characteristics of the signals are affected. Thus, an erroneous logic value when the signal is sampled may result [1].

In this paper, we aim to model the reliability-delay functions of sub-powered interconnects affected by crosstalk effects and process and voltage variations. Our main goal is to determine the probability density functions for switching wires. In the following we'll establish a connection between matrix vector multiplication using Sine, $B-spline$ and Gauss fit methods and an application having as input data a fair column vector *n_Si*, *n_Sp* and *n_Ga* denoting corresponding correct logic values frequencies of appearance in a given division interval as an independent variable and the column vector *MedInt* – dependent variable taken as the reference interval average.

INTERCONNECT SIMULATIONS

We have performed Monte Carlo SPICE simulations for interconnect consisting of 3 wires driven by 3 inverter gates. We have utilized a PTM Resistor (R), Inductor (L), Capacitive (C) model for a local interconnect, while the drivers have been implemented using 45 nm PTM transistor models [8]. The simulated wires have the following dimensions: 50 um length, 70 nm spacing between 2 wires, a 70 nm width, 150 nm thickness and 150 nm height from ground plates. The circuit comprising of the 3 wires driven by the 3 CMOS inverter gates has been supplied at 0.25 V. We have simulated voltage and process variations. The process variations consisted in the variation of RLC parameters. We have applied the process variations only to metal wires and not for the inverter drivers. We have performed Monte Carlo consisting of 5000 simulations for each set of switching input combinations (35 input combinations for 3 wires).

PROPOSED PROBABILITY DENSITY FUNCTIONS MODELING

Prediction Model Using B-Spline Method

Given m real valued t_i, called $knots$, with $t_0 \leq t_1 \leq \cdots \leq t_{m-1}$ a $B-spline$ of degree n is a parametric curve $S: [t_n, t_{m-n-1}] \to \mathbb{R}$ composed of a linear combination of $basis\ B-splines\ b_{i,n}$ of degree n

$$S(t) = \sum_{i=0}^{m-n-2} P_i b_{i,n}(t)\, , t \in [t_n, t_{m-n-1}]$$

The points $P_i \in \mathbb{R}$ are called $control\ points$ or $de\ Boor\ points$. There are $m-n-1$ control points and the convex hull of the control points is a bounding volume of the curve. Note that the first and last n knots lie outside (or equal to the end) of the defined range of the function parameter t [4].

The $m-n-1$ basis $B-splines\ of\ degree\ n$ can be defined, for $n = 0,1, \ldots m-2$, using the Cox-de Boor recursion formula

$$b_{j,0}(t) := \begin{cases} 1\ if\ t_j \leq t < t_{j+1} \\ 0\ otherwise \end{cases}, j = 0, \ldots m-2,\ \ b_{j,n}(t) := \frac{t - t_j}{t_{j+n} - t_j} b_{j,n-1}(t) + \frac{t_{j+n+1} - t}{t_{j+n+1} - t_{j+1}} b_{j+1,n-1}(t),$$

$$j = 0, \ldots, m-n-2$$

Smoothing spline gives $f(x) = piecewise\ polynomial\ computed\ from\ p$, where x is normalized by mean 13.32 and standard deviation 3.174, smoothing parameter $p = 0.9991119$, goodness of fit in **Table 1**.

As prediction method, the function-model stated above has a good fit as expressed by SSE, both R-square evaluation methods and the RMSE as well. The prediction model offers mostly well distributed points where the prediction function has its range with almost the same mean and standard deviation, but having different finite differences between consecutive points than the original vector. The graphic of the function is shown in **Figure 1**.

Prediction model using Sine interpolation method

The band-limited interpolant to δ is the $periodic\ sine\ function\ S_N$: $S_N(x) = \frac{\sin(\pi x/h)}{(2\pi/h)\tan(x/2)}$. Note that since $\tan x/2 \sim x/2$ as $x \to 0$, $S_N(x)$ behaves like the nonperiodic $sine$ function $S_h(x) = (\sin(\pi x/h))/(\pi x/h)$ in the limit $x \to 0$ – independently of h [7].

An expansion of a periodic grid function v in the basis of shifted periodic delta functions takes the form $v_j = \sum_{m=1}^{N} v_m \delta_{j-m}$ in analogy to $v_j = \sum_{m=-\infty}^{\infty} v_m \delta_{j-m}$ Thus the band-limited interpolant of

$$p(x) = \frac{1}{2} \sum_{k=-N/2}^{N/2} e^{ikx_j} \hat{v}_k\, , j = 1, \ldots N$$

(where the terms $k = \pm N/2$ are multiplied by $\frac{1}{2}$), can be written in analogy to $p(x) = \sum_{m=-\infty}^{\infty} v_m S_h(x - x_m)$ as $p(x) = \sum_{m=1}^{N} v_m S_N(x - x_m)$. The general model Sin8 is given by:

$$f(x) = a_1 \sin(b_1 x + c_1) + a_2 \sin(b_2 x + c_2) + a_3 \sin(b_3 x + c_3) + a_4 \sin(b_4 x + c_4) + \\ + a_5 \sin(b_5 x + c_5) + a_6 \sin(b_6 x + c_6) + a_7 \sin(b_7 x + c_7) + a_8 \sin(b_8 x + c_8)$$

Goodness of fit is given in **Table 1**.

As prediction method, the function-model stated above has a good fit as expressed by SSE, both R-square evaluation methods and the RMSE as well. The prediction model offers mostly well distributed points where the prediction function has its range with almost the same mean and standard deviation, but having different finite differences between consecutive points than the original vector. The graphic of the function is shown in Figure 1.

Here the fitness is pursued through the Nonlinear Least Squares using Levenberg-Marquard algorithm with a given 400 maximum of iterations.

Prediction Model Using Gauss Interpolation Method

In curve fitting by a sum of Gaussians the objective is to find the minimum number of Gaussians that can approximate a data set with a prescribed accuracy. When a data set is represented by a sum of Gaussians, a subset containing only very wide Gaussians will generate a coarse representation, and as narrower Gaussians are included, finer representations will be obtained [3].

In one dimension, the Gaussian function is the probability density function of the normal distribution, sometimes also called the frequency curve (see [11]):

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)},$$

General model Gauss8 is given by:

f*(x) = a1*exp(-((x-b1)/c1)^2) + a2*exp(-((x-b2)/c2)^2) + a3*exp(-((x-b3)/c3)^2) + a4*exp(-((x-b4)/c4)^2) + a5*exp(-((x-b5)/c5)^2) + a6*exp(-((x-b6)/c6)^2) + a7*exp(-((x-b7)/c7)^2) + a8*exp(-((x-b8)/c8)^2).*

Goodness of fit is given in **Table 1**.

Fit / Method	SSE	R-square	Adjusted R-square	RMSE
B-Spline	51.55	0.9296	0.8253	0.16
Sine	791.5	-0.08047	-0.08546	0.3988
Gauss	727.5	0.00820	0.003622	0.3824

TABEL 1. Representation of goodness of fit for the given series for the three methods

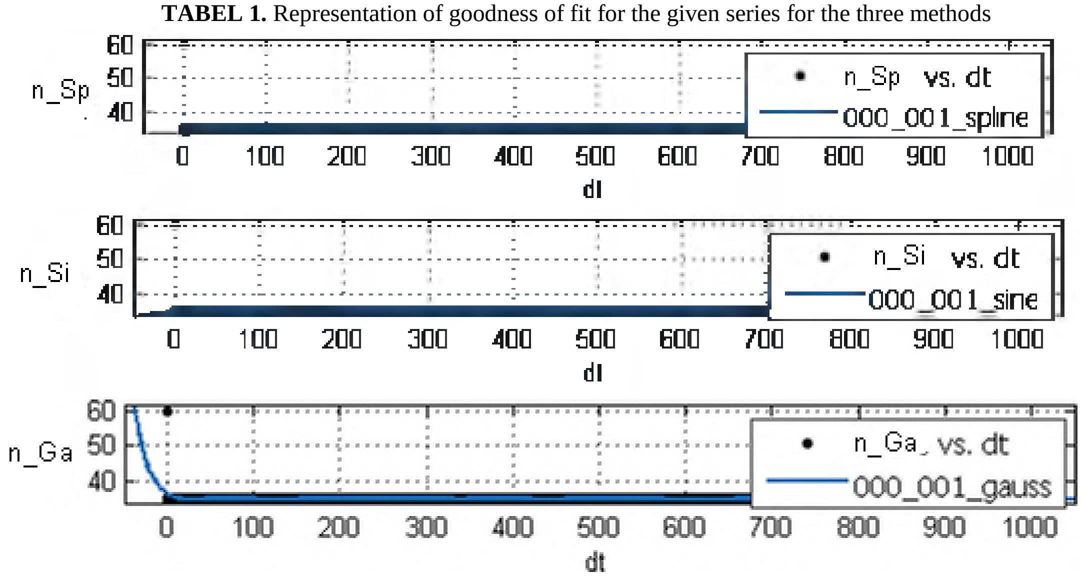

FIGURE 1. Dependence of probability of correctness on the considered delay for the Inverter operating at 25ºC for 0-1 output switching (probabilities on the 2 ns - 5 ns interval) . B-spline, Sine and respectively Gauss interpolated prediction functions which approximate the probabilities of the output switching on the 2 ns - 5 ns interval depending on the average value *MedInt*..

The dots represented in the above figure are the values of the variable *n_Si*, *n_Sp* and *n_Ga* taken over each interval of measurement. The closest functions which could approximate this statistical series are B-spline, Sine and respectively Gauss interpolated prediction functions. Their definition domains consist of the continuous reunion of all measurement intervals, and their values are also in continuous intervals ranging from the minimum value of all intervals to the maximum value of all intervals.

The blue line figured function represents the B-spline, Sine and respectively Gauss interpolated prediction functions which approximate the probabilities of the output switching on the 2 ns - 5 ns interval depending on the average value *MedInt*. Now, for the column vector *n_Si, n_Sp* and *n_Ga* as an independent variable and the column vector *MedInt* – dependent variable as the reference interval average, fit computation did not converge: fitting stopped because the number of iterations or function evaluations exceeded the specified maximum ([5], [10]).

Fit was found when optimization terminated, but as seen below, SSE and RMSE have too big values for the Gauss8 method to be considered a good approximation and prediction method (graph in Figure 1).

CONCLUSIONS

It is possible to establish a ranking among previous selected methods of approximation. As seen by RMSE and SSE values for the selected discrete variables together with their dependencies, as those values drop the better approximation we get. Unfortunately, the known interpolation and approximation methods used above (Gauss, Sine) does not provide good fitting. Spline curves method proved itself the best one in low values for RMSE and SSE, but also for best curve fitting obtained in a minimum number of iterations.

It's worth studying a combination of the above methods for the given data but also to apply this research to a bigger volume of data, thus minimizing error rates and getting less sparse variable values on a greater number of division intervals.

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REFERENCES

1. K. Agarwal, D. Sylvester, D. Blaauw, F. Liu, and S. Vrudhula "*Variational Delay Metrics for Interconnect Timing Analysis*" Proceedings of the Design Automation Conference, pp 381-383, 2004
2. D. Boning, S. Nassif, "*Models of process variations in device and interconnect*", Design of High-Performance Microprocessor Circuits, chapter 06, pp. 98-116, 1999
3. A. Goshtasby, W. D. O'Neill, *Curve Fitting by a Sum of Gaussians*, Graphical Models and Image Processing Vol. 56, No. 4, July, pp. 281-288, 1994
4. H. Helson and D. Lowdenslager, *Prediction theory and Fourier series in several variables*, Acta Mathematica, 99 , 1958, pp. 165-202.
5. S. R. Kadaba, S. B. Gelfand, and R. L. Kashyap, *Recursive estimation of images using non-gaussian autoregressive models*, IEEE Transactions on Image Processing, 7(10), 1998, pp. 1439-1452.
6. N. S. Nagaraj, "*Interconnect process variations: theory and practice*", Proceedings of the 19th International Conference on VLSI Design (VLSID), 2006
7. R. Pachon, P. Gonnet, J. Van Deun, *Fast and Stable Rational Interpolation in Roots of Unity and Chebyshev points*, Oxford University Mathematical Institute Numerical Analysis Group, June 2010.
8. *Predictive Technology Model*, **http://ptm.asu.edu**
9. A. Sanyal, A. Pan, S. Kundu, "*A study on impact of loading effect on capacitive crosstalk noise*" Proc. Int. Symp. On Quality Electronic Design (ISQED), 2009
10. N. Vakhania, *Probability distributions on linear spaces*, North-Holland, 1981
11. Weisstein, Eric W., *Gaussian Function*, MathWorld- A Wolfram Web Resource. http://mathworld.wolfram.com/GaussianFunction.html